When Mindfulness is a problem
and other Unintended consequences of cognitive instruction

Memory, Critical Thinking, Understanding and Problem Solving in Math Education

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My cell phone number (without extension)

remember this

292-3681

Harmful effects of "carrying" and "borrowing"?

Kamii and Dominick, NCTM Yearbook, 1998

An idea common in writings about math education: mental abbreviations for complex operations harms understanding

> *The conventional algorithm makes children treat every column as a column of ones. For example, if we listen to them while they are using the algorithm to solve 136+246, we can hear them saying "Six and six is twelve. Put down the two and carry the one. One and three and four is eight...One and two is three..." Treating every column as a column of ones is convenient for adults, who already know solidly that the "3" in "136" means 30. For young children who are still trying to learn place value, however,* **the conventional algorithms serve to "unteach" place value.**

A common prescription to avoid this problem is to demand terms that make meaning explicit,
EG "composition" and "decomposition"

The harmful effects of not "carrying" and "borrowing"

- Early years introduction to arithmetic serves to inculcate life-long habits of thought and practice.
- Early arithmetical procedures should establish practices which can be carried out with minimal effort without creating complex demands on working memory
- Training recitation of explicit meaning at every stage clutters the process it anchors attention to low-level detail
- Algorithms are processes; while meaning is important and should be established it should not add drag to execution.
- Kamii and Dominick acknowledge that adults have no difficulty interpreting place value distinctions. Presumably they refer to adults who learned ... the standard algorithms with "borrowing" and "carrying"
- This practice drags activities out: tedious and cluttered.

The point of place value

- Transitioning to place-value arithmetic effected a revolution in mathematical power which can be credited in part with powering the scientific and instustrial revolutions.
- The ancients were not numerical slouches. But they represented different scales of number explicitly in different ways: I, X, C, M, etc.
- Adding $100 + 100$ involved one process with one meaning: $C + C$; adding 10s or 1s meant something different.
- The key to place value representation is the sameness of these operations at every scale; in every "position"
- The demand to make distinctions of meaning explicit effectively erases place value. "We murder to dissect" –Wordsworth.

What was my cell phone number?

Someone who didn't write it down answer!

Answer: 292-3681

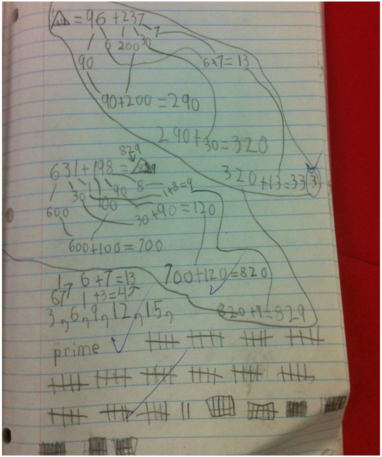
Now remember my office and home numbers:

Office: 474-7489

Home: 261-3532

Explicit meanings revert to Roman numeral arithmetic?

A recent example:



Commonly place value is eliminated by the placement of information arbitrarily on the page. While digital representation is retained (200 v CC). The point is to focus on the "value" distinction as separate from the "place".

Again, the point of place value is to "store" meaning in position. so that mental representation is reduced to handling single symbols in a single column at all times.

The rule of seven

(plus or minus 2)

So ... who remembers my work phone number?

474-7489

My home phone number?

261-3532

Cognitive psychologists speak of a *rule of seven*:

An average human can hold about 7 ± 2 items (of abstract information i.e., symbols) in working memory.

Origin: Miller, G. A., *The magical number seven, plus or minus two: Some limits on our capacity for processing information* Psychological Review **63** (2): 1956, 81–97

...one of most frequently cited papers in psychological research

– also sometimes referred to as **Miller's Law**.

How can we think, despite this rule?

Can we cheat short term memory?

Cognitive science also tells us that short term memory is critical in all higher cognitive processes:

critical thinking

decision making

problem solving

reflection

creativity

etc.

Since we can only juggle a tiny bit of symbolic information there, how can our brains perform these higher tasks?

Experientially: obvious that **juggling information** (like phone numbers or abstract symbols) **burdens our conscious thought and degrades higher functions**.

(Detail-) Mindfulness

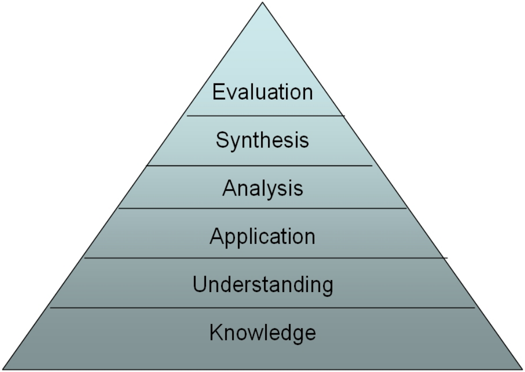
A seductive (for mathematicians) but bad idea

(Detail-)mindfulness (my tentative term to capture): the idea students should develop a habit of thinking about the details of mathematics rather than performing them automatically.

- A seductive idea for mathematicians because
  - Nobody cares more about students understanding math
  - It is easy to conflate with having understanding
  - Or to believe it will enhance long-term understanding
- But when inculcated as a *permanent habit* mindfulness
  - clutters thinking
    by filling working memory with elementary details
  - impairing higher-order thinking
  - overloads working memory
  - directs attention to meaning at the wrong level

“Bloom”ing idiots**?

Building a house from the roof down*



This approach appears to be based on the idea that attention to the higher-order thinking skills (creativity, critical thinking etc) obviates the priority of establishing the lower ones.

In the extreme they are regarded as antithetical.

** H A Doughty, *Blooming Idiots: Ed. Obj, learning taxonomies ...* Coll. Quarterly (2006)

* M J Booker, *A roof without walls: Bloom’s Taxonomy & misdirection of Am. Educ.* Acad. Quest (2007)

Facts Prevent Understanding?

(Daisy Christodoulou's Myth # 1)

"Rousseau, Dewey and Freire...were **hostile to fact-learning**, ... set up a **dichotomy between facts and true understanding**...the rhetoric of the current [UK] national curriculum ... is based on a similar ... opposition between facts and understanding.
Our long-term memories are capable of storing a great deal of information whereas our working memories are limited. Therefore, it is very important that we do commit facts to long-term memory, as **this allows us to 'cheat' the limitations of working memory**. The facts we've committed to memory help us to understand the world and to solve problems."

– D. Christodoulou, *Seven Myths about Education*, Routledge (2014)

Another challenge

Memorize the following string of 35 letters and spaces

QDR CCBO PLHBMDTE ETR XUVI HHBWDYC

Not easy? How about the following 35?

MY CAR IS THE WHITE ONE ON THE LEFT

Why is the second easier to memorize than the first?

New information is easier to maintain in short-term memory (and to commit to long-term memory) when it attaches to information already in long-term memory.

Observe that access to long-term memory to attach new information is **instantaneous** and **effortless**.

The "constructivist teaching fallacy"

Constructivism—(descriptive) theory of learning:

New knowledge is acquired by attaching it to knowledge already in long-term memory, adding onto existing body of knowledge.

Accordingly, the more information you have in long-term memory, the easier it is to acquire new information.

Certain (prescriptive) theories of teaching going by this name are **based on the idea that children learn most effectively when given less guidance and direct instruction**, and find things out for themselves

"Many educators ... have assumed that the best way to promote [knowledge] construction is to have students try to discover new knowledge or solve new problems without explicit guidance from the teacher. **Unfortunately, this assumption is both widespread and incorrect. Mayer calls it the 'constructivist teaching fallacy.'**" – Kirshner, Sweller, Clark, "Putting students on the path to learning" *American Educator (2012).*

Dan Willingham: Can you get by with mental skills

and just "look up" information As needed?

" 'thinking' [is] combining information in new ways...In today's world, is there a reason to memorize anything? You can find any factual information you need in seconds via the Internet....[i.e., eliminating need for memory]

...this argument is false. Data from the last 30 years lead to a **conclusion that is not scientifically challengeable: thinking well requires knowing facts**, and that's true not simply because you need something to think about. The very processes that teachers care about most—**critical thinking processes like reasoning and problem solving—are intimately intertwined with factual knowledge ... in long-term memory**

...to think critically about, say, ... the Second World War, that does not mean that we can think critically about a chess game, or about the current situation in the Middle East, or even about the start of the American Revolutionary War. The **critical thinking processes are tied to the background knowledge.**

Facts versus understanding in Canadian educ. system

This false dichotomy often arises in our meetings with ministry officials and math (education) consultants in Manitoba.

- “They shouldn’t memorize math facts until they’ve understood”

- “I would let a child use long division only after they can show understanding of it in three different ways”

- “If you teach them the standard algorithm they will prefer it to strategies” (which are considered to “show understanding”)

- “If they see how before they know why they’ll never understand”

- Memorization demagogued as “rote” (“without understanding”*)

- Pejoratives like “Drill and kill” (better: “drill for the thrill of skill”!)

*Not the actual meaning—rote means “by repetition”

Effect on WNCP (and ONT) curriculum

> "By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development."
> – WNCP K-8 Framework

No outcomes referencing or necessitating memorization of (single digit) math facts.

All four standard algorithms are absent (by design)
- Adding/subtracting by stacking, with carry/borrow
- Multiplying vertically
- Long division

In 2013 MB and in 2014 AB returned memorization of math facts to their provincial curricula. MB also now mentions standard algorithms
These things remain absent in curricula in 8 provinces.

Yes, standard algorithms used to be mentioned!

MB curriculum 1978–1995

6. Multiplication Algor...

The student should be able to:

– demonstrate the use of the standard algorithm using digits up to a combined total of eight (e.g., 4 digits by 4 digits, 2 digits by 6 digits, etc.)

Developing the algorithm:

a. Stress basic facts.

b. Stress estimation activities.

c. Stress earlier concepts, i.e., place value, addition.

```
  653
×   8
   24    (ones) 8 × 3
  400    (tens) 8 × 50
 4800    (hundreds) 8 × 600
 5224
```

The "*standard algorithm*" should be the goal of both teacher and student, as it is the most efficient.

```
     653
×     18
   5 224   (8 × 653)
   6 530   (10 × 653)
  11 754
```

```
     653 \
×     18  |
   5 224  } ⟶ Students who have no difficulty understanding place value could use this form.
   6 53   |
  11 754 /
```

CHMENT: Multiply using some recognized non-standard algorithms.

Non-standard algorithms often provide children with motivating activities, particularly when they are related to history. There are a number of books now available to provide teachers with backgrounds.

e.g., lattice multiplication, Napier's Bones, Russian peasant method, doubling method, etc.

“But ‘math facts’ are already in there!”

Gr 5 outcomes (AB 2014 changes)

Apply mental mathematics strategies and number properties, such as:

- skip counting from a known fact
- using doubling or halving
- using patterns in the 9s facts
- using repeated doubling or halving

in order to understand and recall basic multiplication facts (multiplication tables) to 81 and related division facts.
[C, CN, ME, R, V]

Understand, recall and apply multiplication and related division facts to 9×9.

- ➢ Describe the mental mathematics strategy used to determine a given basic fact, such as:
  - skip count up by one or two groups from a known fact; e.g., if $5 \times 7 = 35$, then 6×7 is equal to $35 + 7$ and 7×7 is equal to $35 + 7 + 7$
  - skip count down by one or two groups from a known fact; e.g., if $8 \times 8 = 64$, then 7×8 is equal to $64 - 8$ and 6×8 is equal to $64 - 8 - 8$
  - doubling; e.g., for 8×3 think $4 \times 3 = 12$, and $8 \times 3 = 12 + 12$
  - patterns when multiplying by 9; e.g., for 9×6, think $10 \times 6 = 60$, and $60 - 6 = 54$; for 7×9, think $7 \times 10 = 70$, and $70 - 7 = 63$
  - repeated doubling; e.g., if 2×6 is equal to 12, then 4×6 is equal to 24 and 8×6 is equal to 48
  - repeated halving; e.g., for $60 \div 4$, think $60 \div 2 = 30$ and $30 \div 2 = 15$.
- ➢ Explain why multiplying by zero produces a product of zero (zero property of multiplication).
- ➢ Explain why division by zero is not possible or is undefined; e.g., $8 \div 0$.
- ➢ Determine, with confidence, answers to multiplication facts to 81and related division facts.
- ➢ Demonstrate understanding, recall/memorization and application of multiplication and related division facts to 9×9.

- “recall” added, muted – an afterthought
- **Workarounds** – to compensate for not committing to heart
- Knowing a workaround $\neq$ “understanding”!
- Best educ. systems require mastery to 9×9 in Gr 3 or 4.

Mindfulness can be the enemy of progress in math

"**It is a profoundly erroneous truism ... that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them.**"

- Alfred North Whitehead

Project Follow Through (PFT)

The most ironically-named education study ever

(Also the **largest** and **most expensive** study of its kind ever!): over 700,000 children over 10 years, starting in 1968

Findings: Direct Instruction (DI) – explicitly taught, followed by practice, feedback assessment resulted in students who had:

- Better basic skills
- Better self-esteem
- Better understanding
- Better problem-solving skills

The only instructional model that consistently improved basic skills, understanding and self-esteem was DI.

DI is a bottom-up approach

Higher-order thinking, problem-solving skills, self-esteem all arise from mastery of foundational skills.

Project Follow Through: Failures

What about instructional methods that stress "higher-order" cognitive skills –creativity, problem-solving etc?

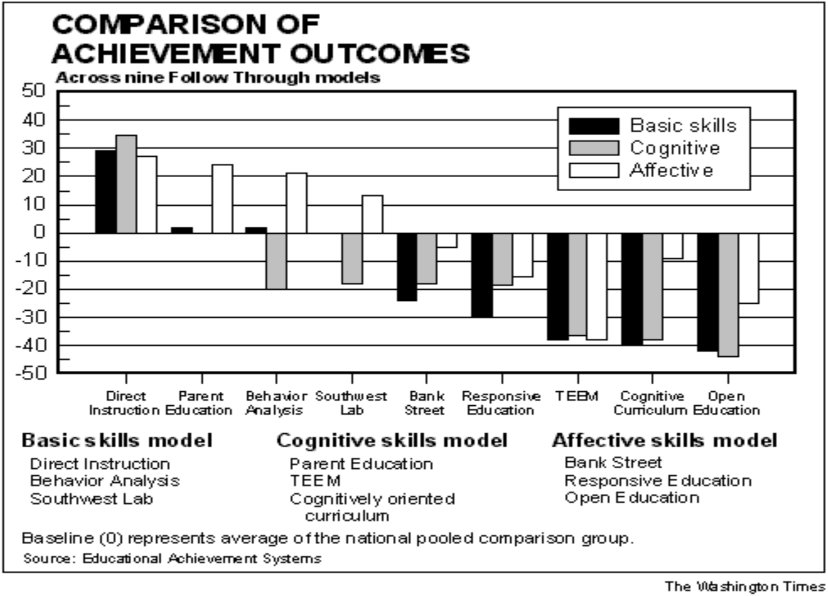
I.E.: top-down approach assuming that basic skills, self-esteem arise automatically when struggling with complex problems?

Models explicitly teaching cognitive skills negatively impacted those skills relative to the control[1]. The focus on top-down learning is based upon a false assumption.

One of the worst, the "High/Scope Cognitively Oriented Curriculum" shares many traits with "WNCP" and "21st Century Learning" models. It is a direct precursor of the most popular discovery-based learning systems of today.

[1] "Control" = comparable groups taught with no intervention

Project Follow Through: Results



Observations

Skills vs understanding = false dichotomy

Direct instruction vs understanding = false dichotomy

There is an increase in students lacking both understanding and skill–obsession with understanding is not working (misunderstandings about understanding)

Q: Is "understanding" possible without basic facts & skills?

Trying to teach higher-level skills (top of Bloom's) without lower ones in place undermines **both**!

A habit of always focussing on lowest-level details does not enhance understanding – it impairs it.

Finally...

Without a broadly taught foundation students lack the tools to advance

If I have seen further than others, it is because I have stood on the shoulders of giants

- Sir Isaac Newton

THANK YOU FOR LISTENING!

Some resources

Giving broad, solid foundations—available for use in schools immediately

JUMP Math: jumpmath.org

Saxon Math (Heritage Resources, Carmen, MB)

Singapore Math (Heritage Resources, Carmen, MB)

Note: JUMP Math is now a Manitoba Recommended Resource; teachers can use it in the classroom ... *because we asked.*